CBSC A-Level Mathematics Induction Booklet

Preparing students for the challenge of A Level Mathematics

Mathematics Transition to A Level

Overview of A level Mathematics
Your Year 12 A Level Mathematics course will consist of 'Pure Mathematics' and 'Mechanics and Statistics' for which you will need two textbooks. Each book is written by senior examiners and includes a unique access code to access your ActiveBook.

The cost of each book is approximately £20.00.
Each one will be available to purchase from the Mathematics Department in September 2018 for £20.00 each.

You will also require a scientific calculator – Model CASIO fx991EX
The cost of the scientific calculator is approximately £30.00 (price as quoted from an online resource) but will be available at a discounted price from the Mathematics department in September 2018 for £20.00 also.

Pupils can choose to keep their books and/or calculator for the duration of their course, or return them at the end of the course in the same condition they received them and have their cost refunded.

A Level mathematics uses many of the skills you developed at GCSE. The big difference is that you will be expected to recognise where you use these skills and apply them quickly and efficiently.

In order to get off to a good start you need to be prepared. This booklet will help you get ready for Y12 Mathematics.

This work is compulsory for all prospective A level students.
Some of the websites that you can use to help you with this work are given below:

https://vle.mathswatch.co.uk/vle

https://corbettmaths.com/

https://hegartymaths.com/

https://www.examsolutions.net/

Websites that can help you with A Level topics

https://vle.mathswatch.co.uk/vle/

http://www.examsolutions.net/

http://www.drfrostmaths.com/resources/sow.php?year=A%20Level

https://hegartymaths.com/

https://www.khanacademy.org/

http://www.physicsandmathstutor.com/

www.markit.education

https://www.madasmaths.com/archive_iygb_practice_papers_c4_practice_papers.html

CONTENTS

Make sure you show all the **relevant working out** as you are working through the topics. There are answers at the end of the Booklet to check and correct your working.

You will be assessed on this Grade 7 - 9 work at the start of the term and you need to gain at least 70% to pass.

Make sure you complete the work on separate pieces of paper and make sure you organise the work in the order of the sections and questions. Good organisations skills is a vital part of doing the A-Level Maths course

Section 1: Linear Expressions and Equations

Watch out!
When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '–'.

1.Expand and Simplify where possible

a) $3(2x - 1)$

b) $-(3xy - 2y^2)$

c) $9(3s + 1) - 5(6s - 10)$

d) $2(4x - 3) - (3x + 5)$

e) $4p(2p - 1) - 3p(5p - 2)$

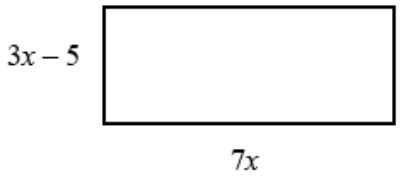
f) $3b(4b - 3) - b(6b - 9)$

g) $\frac{1}{2}(2y - 8)$

h) $13 - 2(m + 7)$

i) $5p(p^2 + 6p) - 9p(2p - 3)$

j) The diagram shows a rectangle.
Write down an expression, in terms of x, for the area of the Rectangle.
Show that the area of the rectangle can be written as $21x^2 - 35x$



2. Factorise.

Hint
Take the highest common factor outside the bracket.

a) $6x^4y^3 - 10x^3y^4$

b) $21a^3b^5 + 35a^5b^2$

c) $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

3. Solve these Equations

a) $8 - (x + 3) = 4$

b) $14 - 3(2x + 3) = 2$

c) $\frac{1}{2}(x + 3) = 5$

d) $\frac{2x}{3} - 1 = \frac{x}{3} + 4$

e) $\frac{y}{4} + 3 = 5 - \frac{y}{3}$

f) $\frac{7x - 1}{2} = 13 - x$

g) $\frac{y - 1}{2} + \frac{y + 1}{3} = \frac{2y + 5}{6}$

4. Solve these simultaneous equations using the **elimination method**.

a) $3x + y = 7$
$3x + 2y = 5$

b) $2x + y = 11$
$x - 3y = 9$

c) $2x + 3y = 11$
$3x + 2y = 4$

5. Solve these simultaneous equations using the **substitution method.**

a) $y = 2x - 3$
$5x - 3y = 11$

b) $2x = y - 2$
$8x - 5y = -11$

c) $3y = 4x - 7$
$2y = 3x - 4$

d) $3x + 2y + 1 = 0$
$4y = 8 - x$

6. Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

7. Solve these pairs of simultaneous equations **graphically.**

a) $y = 3x - 1$ and $y = x + 3$

b) $x + y = 0$ and $y = 2x + 6$

c) $4x + 2y = 3$ and $y = 3x - 1$

d) $2x + y + 4 = 0$ and $2y = 3x - 1$

Section 2 Quadratic Expressions and Equations

1. Expand and simplify.

a) $(x + 7)(x - 2)$ b) $(5x - 3)(2x - 5)$ c) $(3x + 4y)(5y + 6x)$

d) $(4x - 3y)^2$ e) $(x + 3)^2 + (x - 4)^2$ **f)** $\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$ **g)** $\left(x+\frac{1}{x}\right)^2$

2. Factorise

a) $x^2 + 7x + 12$ b) $x^2 - 11x + 30$ **c)** $x^2 - 7x - 18$

d) $36x^2 - 49y^2$ e) $18a^2 - 200b^2c^2$ h) $2x^2 + x - 3$

3. 4Simplify the algebraic fractions.

a) $\frac{2x^2+4x}{x^2-x}$ b) $\frac{x^2-2x-8}{x^2-4x}$ c) $\frac{x^2-5x}{x^2-25}$

d) $\frac{2x^2+14x}{2x^2+4x-70}$ e) $\frac{9x^2-16}{3x^2+17x-28}$ f) $\frac{2x^2-7x-15}{3x^2-17x+10}$

g) Simplify $\sqrt{x^2+10x+25}$

h) Simplify $\frac{(x+2)^2+3(x+2)^2}{x^2-4}$

4. Solving quadratic equations

Solve these equations by **factorising**

a $6x^2 + 4x = 0$ **b** $x^2 - 3x - 4 = 0$ **c** $2x^2 - 7x - 4 = 0$

d $x^2 - 3x = 10$ e $x(x + 2) = 2x + 25$ f $x(3x + 1) = x^2 + 15$

Solve these equations **by completing the square**

a $x^2 - 4x - 3 = 0$
b $x^2 + 8x - 5 = 0$
c $2x^2 + 8x - 5 = 0$
d $(x - 4)(x + 2) = 5$
e $2x^2 + 6x - 7 = 0$

Hint
Get all terms onto one side of the equation.

Solve these equations **by formula**

a) Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

b) Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

Hint
Get all terms onto one side of the equation.

Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a) $4x(x - 1) = 3x - 2$

b) $x(3x - 1) = 10$

5. Quadratic Simultaneous Equations

a) $y = x - 3$
$x^2 + y^2 = 5$

b) $y = 3x - 5$
$y = x^2 - 2x + 1$

c) $y = 2x$
$y^2 - xy = 8$

d) $x - y = 1$
$x^2 + y^2 = 3$

6. Solve these pairs of simultaneous equations **graphically**.

a) $y = x - 1$ and $y = x^2 - 4x + 3$

b) $y = 1 - 3x$ and $y = x^2 - 3x - 3$

c) $x + y = 1$ and $x^2 + y^2 = 25$

Section 3 Inequalities

Solve these **linear inequalities**.

a) $1 \geq 3x + 4$ b) $5 - 2x < 12$ c) $8 < 3 - \frac{x}{3}$

d) $3 \leq 7x + 10 < 45$ e) $3(2 - x) > 2(4 - x) + 4$ f) $5(4 - x) > 3(5 - x) + 2$

g) Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$

Quadratic Inequalities

a) Find the set of values of x for which $(x + 7)(x - 4) \leq 0$

b) Find the set of values of x for which $x^2 - 4x - 12 \geq 0$

c) Find the set of values of x for which $12 + x - x^2 \geq 0$

d) Find the set of values which satisfy the following inequality $x(2x - 9) < -10$

e) Find the set of values which satisfy the following inequality $6x^2 \geq 15 + x$

Section 4

Equation of Straight Lines

1. Find the gradient and the y-intercept of the following equations.

a) $y = -\frac{1}{2}x - 7$ b) $x + y = 5$ c) $2x - 3y - 7 = 0$

2 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

a) gradient $-\frac{1}{2}$, y-intercept -7 b) gradient 2, y-intercept 0

c) gradient $\frac{2}{3}$, y-intercept 4

3 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

4 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

5 Write an equation for the line passing through each of the following pairs of points. **A** (4, 5), (10, 17)

6 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Parallel and Perpendicular Lines

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ $(3, 2)$ b $2x + 4y + 3 = 0$ $(6, -3)$

Hint

If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

2 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ $(4, 0)$ **b** $x - 4y - 4 = 0$ $(5, 15)$

3 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a $(4, 3)$, $(-2, -9)$ **b** $(0, 3)$, $(-10, 8)$

4 Work out whether these pairs of lines are parallel, perpendicular or neither.

d $3x - y + 5 = 0$ $x + 3y = 1$ **e** $2x + 5y - 1 = 0$ $y = 2x + 7$ **f** $2x - y = 6$ $6x - 3y + 3 = 0$

5 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

a) Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates $(-8, 3)$.

b) Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin. c) Find an equation of $\mathbf{L_3}$.

Section 5 Sketching Graphs

Quadratic Graphs

1 Sketch the graph of $y = -x^2$.

2 Sketch each graph, labelling where the curve crosses the axes.

a) $y = (x + 2)(x - 1)$ b) $y = x(x - 3)$ c) $y = x^2 - 5x + 4$

d) $y = x^2 + 4x$ e) $y = 9 - x^2$ f) $y = 2x^2 + 5x - 3$,

3 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

a) $y = x^2 - 5x + 6$ b) $y = -x^2 + 7x - 12$ c) $y = -x^2 + 4x$

Cubic and Reciprocal Graphs

4 Here are six equations.

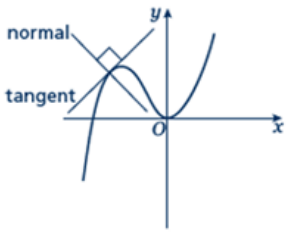
A $y = \frac{5}{x}$ **B** $y = x^2 + 3x - 10$ **C** $y = x^3 + 3x^2$

D $y = 1 - 3x^2 - x^3$ **E** $y = x^3 - 3x^2 - 1$ **F** $x + y = 5$

Hint
Find where each of the cubic equations cross the y-axis.

Here are six graphs.

i



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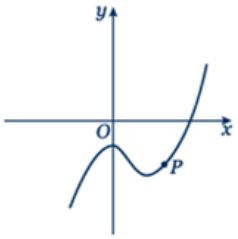

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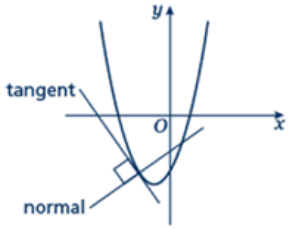

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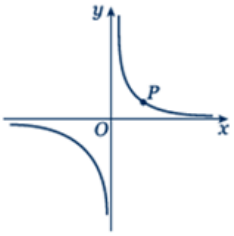

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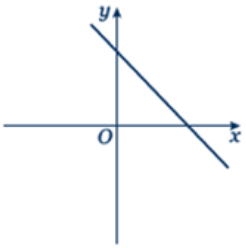

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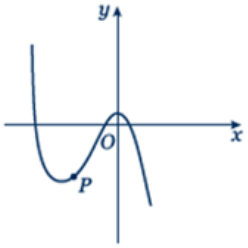


a Match each graph to its equation.

b Copy the graphs ii, iv and vi and draw the tangent and normal each at point P.

5 Sketch the following graphs

a) $y = 2x^3$

b) $y = x(x-2)(x+2)$

c) $y = (x+1)(x+4)(x-3)$

d) $y = (x-3)^2(x+1)$

e) $y = \frac{3}{x}$

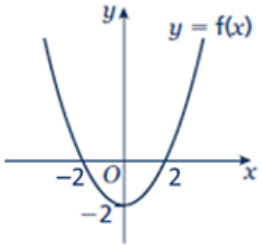
f) $y = -\frac{2}{x}$

g) Sketch the graph of $y = \frac{1}{x+2}$

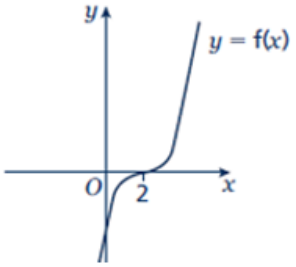
h) Sketch the graph of $y = \frac{1}{x-1}$

Transformations of Graphs

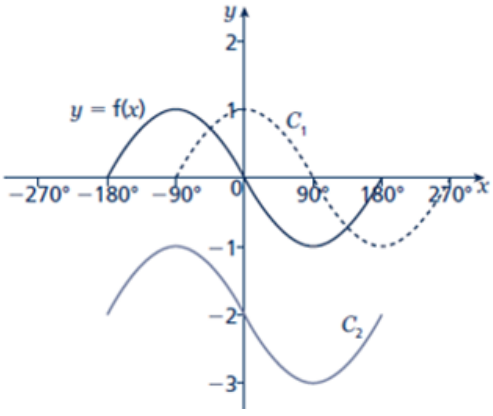
6 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x) + 4$ and $y = f(x+2)$.



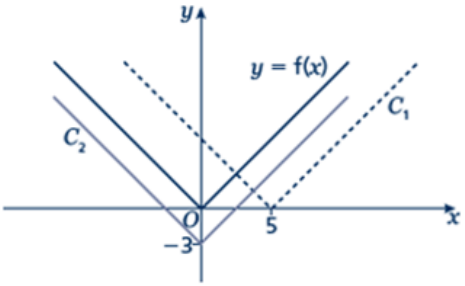
7 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x+3)$ and $y = f(x) - 3$.



8 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2.
Write down the equations of the translated curves C_1 and C_2 in function form.



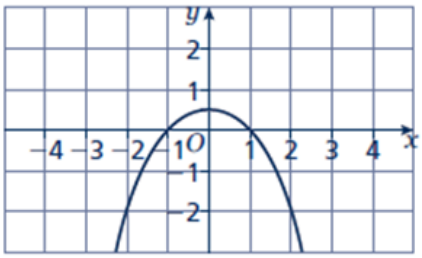
9 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



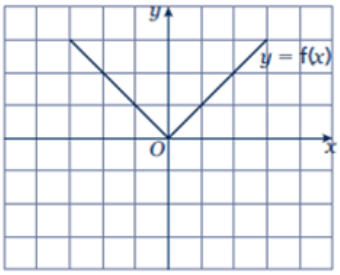
10 The graph shows the function $y = f(x)$.

a Sketch the graph of $y = f(x) + 2$

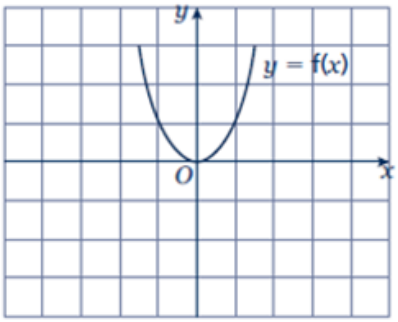
b Sketch the graph of $y = f(x + 2)$



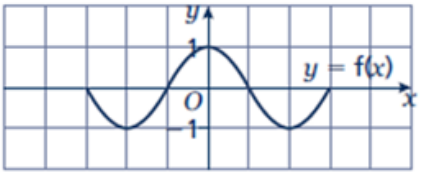
11 The graph shows the function $y = f(x)$. Copy the graph and on the same axes sketch and label the graphs of $y = -2f(x)$ and $y = f(3x)$.



12 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch and label the graphs of $y = -f(x)$ and $y = f\left(\frac{1}{2}x\right)$.



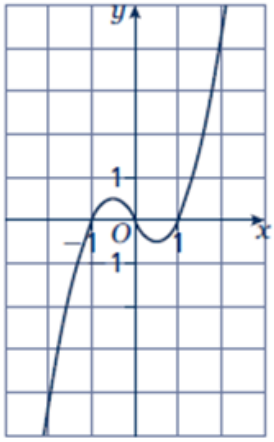
13 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch the graph of $y = -f(2x)$.



14 The graph shows the function $y = f(x)$.

a Sketch the graph of $y = -f(x)$.

b Sketch the graph of $y = 2f(x)$.



15 **a** Sketch and label the graph of $y = f(x)$, where $f(x) = (x - 1)(x + 1)$.

b On the same axes, sketch and label the graphs of $y = f(x) - 2$ and $y = f(x + 2)$.

16 **a** Sketch and label the graph of $y = f(x)$, where $f(x) = -(x + 1)(x - 2)$.

b On the same axes, sketch and label the graph of $y = f\left(-\frac{1}{2}x\right)$.

Section 6 Proportion

1 Paul gets paid an hourly rate. The amount of pay (£P) is directly proportional to the number of hours (h) he works.
When he works 8 hours he is paid £56.
If Paul works for 11 hours, how much is he paid?

2 Q is directly proportional to the square of Z.
$Q = 48$ when $Z = 4$.

a Find a formula for Q in terms of Z.

b Sketch the graph of the formula.

c Find Q when $Z = 5$.

d Find Z when $Q = 300$.

3 m is proportional to the cube of n.
$m = 54$ when $n = 3$.
Find n when $m = 250$.

4 s is inversely proportional to t.

a Given that $s = 2$ when $t = 2$, find a formula for s in terms of t.

b Sketch the graph of the formula.

c Find t when $s = 1$.

5 y is inversely proportional to the square root of x.
$x = 25$ when $y = 1$.
Find x when $y = 5$.

6 a is inversely proportional to b.
$a = 0.05$ when $b = 4$.

a Find a when $b = 2$.

b Find b when $a = 2$.

Section 7 Indices and Surds

1 Evaluate.

a 14^0 **b** $64^{\frac{1}{3}}$ **c** $49^{\frac{3}{2}}$ **d** 6^{-2}

2 Simplify.

a $\dfrac{3x^2 \times x^3}{2x^2}$ c $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$

b $\dfrac{\left(2x^2\right)^3}{4x^0}$ **d** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Watch out!
Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

3 Evaluate.

a $27^{-\frac{2}{3}}$ b $16^{\frac{1}{4}} \times 2^{-3}$ c $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$

4 Write the following as a single power of x.

a $\sqrt[5]{x^2}$ b $\dfrac{1}{\sqrt[3]{x}}$ c $\dfrac{1}{\sqrt[3]{x^2}}$

5 Write the following without negative or fractional powers.

a x^{-3} b $x^{\frac{2}{5}}$ **c** $x^{-\frac{3}{4}}$

6 Write the following in the form ax^n.

a $5\sqrt{x}$ **b** $\dfrac{2}{x^3}$ **c** $\dfrac{1}{3x^4}$

d $\dfrac{2}{\sqrt{x}}$ **e** $\dfrac{4}{\sqrt[3]{x}}$ **f** 3

7 Write as sums of powers of x.

a $\dfrac{x^5+1}{x^2}$ **b** $x^2\left(x+\dfrac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\dfrac{1}{x^3}\right)$

8 Simplify.

a $\sqrt{72}+\sqrt{162}$ b $\sqrt{75}-\sqrt{48}$ **c** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

9 Expand and simplify. **a)** $(4-\sqrt{5})(\sqrt{45}+2)$ b) $(5+\sqrt{2})(6-\sqrt{8})$
c) $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

10 Rationalise and simplify, if possible.

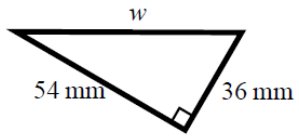
a $\frac{2}{\sqrt{7}}$ b) $\frac{\sqrt{8}}{\sqrt{24}}$ c) $\frac{\sqrt{5}}{\sqrt{45}}$

d) $\frac{2}{4+\sqrt{3}}$ e) $\frac{6}{5-\sqrt{2}}$ **f)** $\frac{1}{\sqrt{9}-\sqrt{8}}$ **g)** $\frac{1}{\sqrt{x}-\sqrt{y}}$

Section 8 Pythagoras and Trigonometry

1 Work out the length of the unknown side in each triangle.
Give your answers in **surd form.**

a



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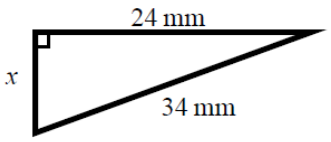

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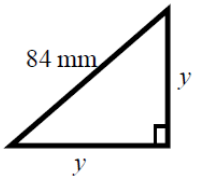
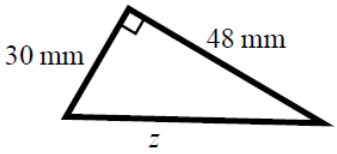


d



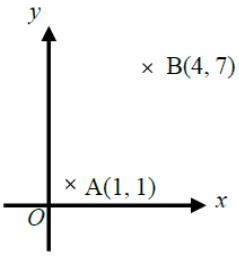
2 A rectangle has length 84 mm and width 45 mm.
Calculate the length of the diagonal of the rectangle.
Give your answer correct to 3 significant figures.

Hint
Draw a sketch of the rectangle.

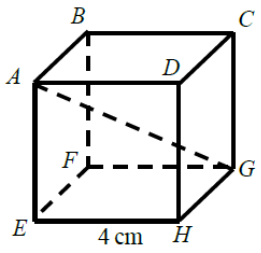
3 A yacht is 40 km due North of a lighthouse.
A rescue boat is 50 km due East of the same lighthouse.
Work out the distance between the yacht and the rescue boat.
Give your answer correct to 3 significant figures.

Hint
Draw a diagram using the information given in the question.

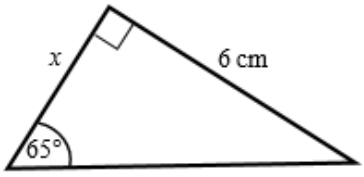
4 Points A and B are shown on the diagram.
Work out the length of the line AB.
Give your answer in surd form.



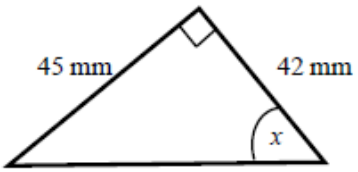
5 A cube has length 4 cm.
Work out the length of the diagonal *AG*.
Give your answer in surd form.



6 Calculate the length of the unknown side in the triangle to 3 s.f

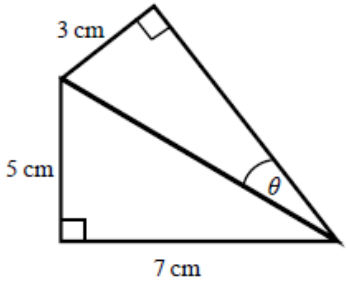


7 Calculate the size of angle x in the triangle to 1 decimal place.



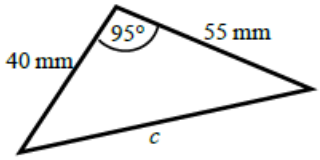
8 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

Hint:
First work out the length of the common side to both triangles, leaving your answer in surd form.

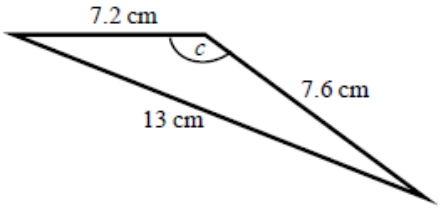


Cosine Rule

9a Find the value of ***C*** to 3s.f

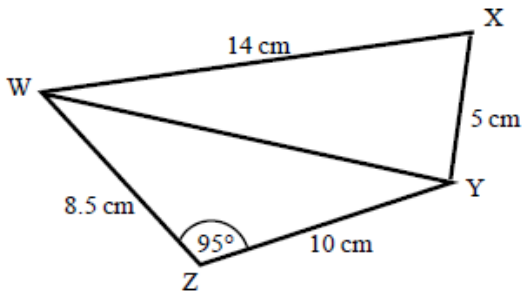


b. Find the value of ***angle C*** to 1 d.p



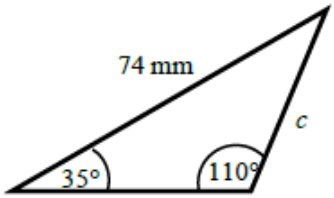
10 **a** Work out the length of WY.
Give your answer correct to
3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to
1 decimal place.

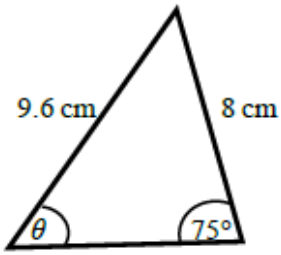


Sine Rule

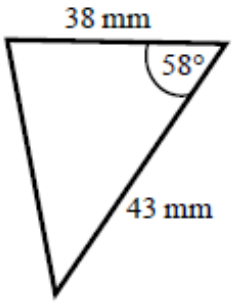
11) Find the value of ***C*** to 3s.f



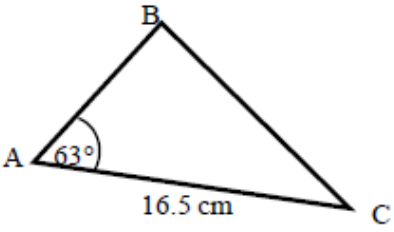
12) Find the value of Θ to 3s.f



13) Find the area of the triangle below

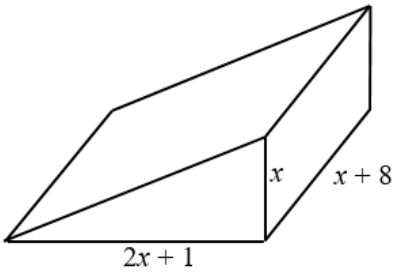


14) The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significant figures.

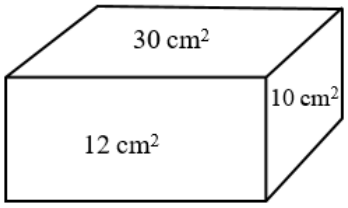


Section 9 Area and Volume

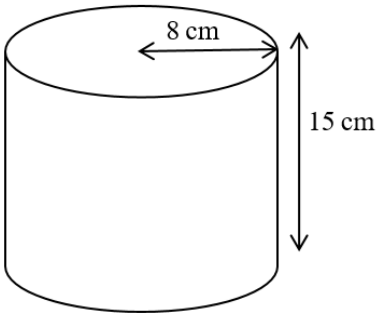
1 The diagram shows a solid triangular prism.
All the measurements are in centimetres.
The volume of the prism is V cm^3.
Find a formula for V in terms of x.
Give your answer in simplified form.



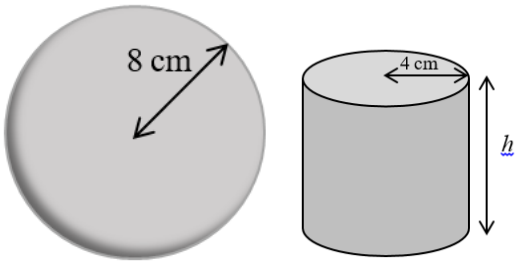
2 The diagram shows the area of each of three faces of a cuboid.
The length of each edge of the cuboid is a whole number of centimetres.
Work out the volume of the cuboid.



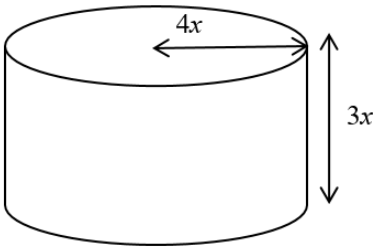
3 The diagram shows a large catering size tin of beans in the shape of a cylinder.
The tin has a radius of 8 cm and a height of 15 cm.
A company wants to make a new size of tin.
The new tin will have a radius of 6.7 cm.
It will have the same volume as the large tin.
Calculate the height of the new tin.
Give your answer correct to one decimal place.



4 The diagram shows a sphere and a solid cylinder.
The sphere has radius 8 cm.
The solid cylinder has a base radius of 4 cm and a height of h cm.
The total surface area of the cylinder is half the total surface area of the sphere.
Work out the ratio of the volume of the sphere to the volume of the cylinder.
Give your answer in its simplest form.



5 The diagram shows a solid metal cylinder.
The cylinder has base radius $4x$ and height $3x$.
The cylinder is melted down and made into a sphere of radius r.
Find an expression for r in terms of x.

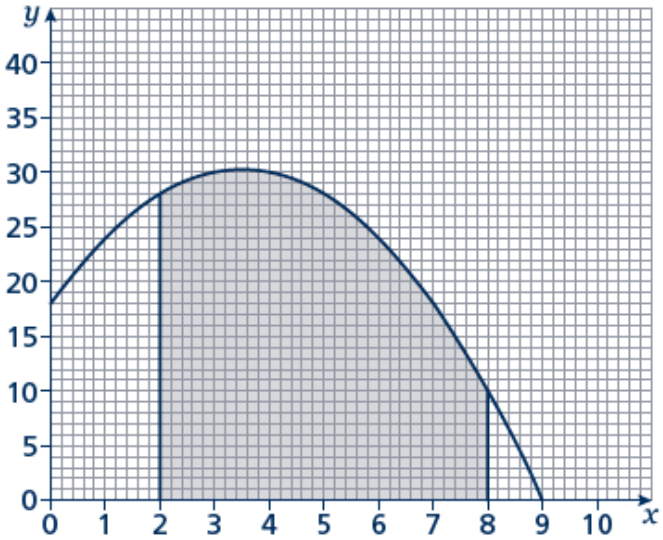


Area under graph

1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x-axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

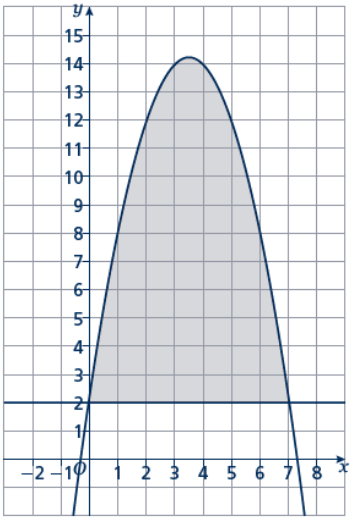
Hint:
For a full answer, remember to include 'units^2'.

2 Estimate the shaded area shown on the axes.
Use six strips of width 1 unit.

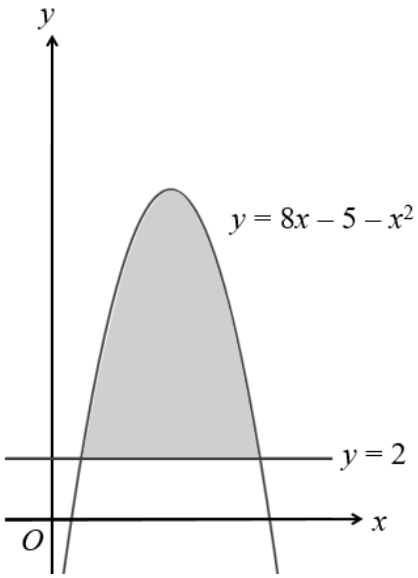


3) Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the x-axis from $x = 2$ to $x = 5$.
Use six strips of equal width.

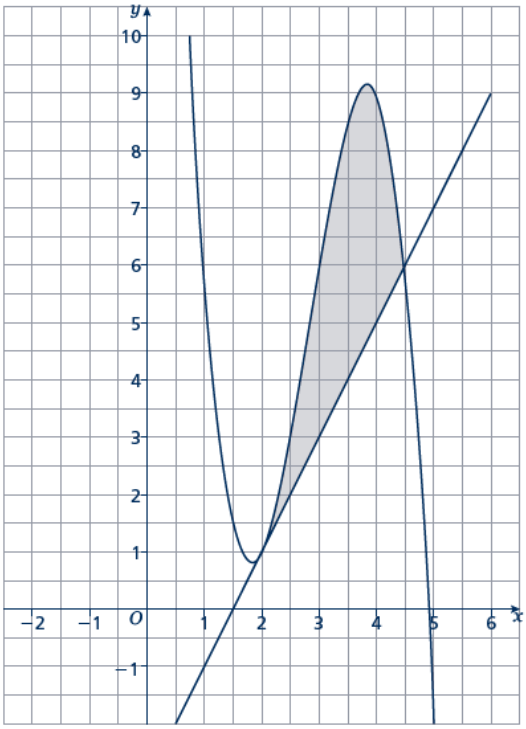
4) Estimate the shaded area.
Use seven strips of equal width.



5) The curve $y = 8x - 5 - x^2$ and the line $y = 2$ are shown in the sketch.
Estimate the shaded area using six strips of equal width.



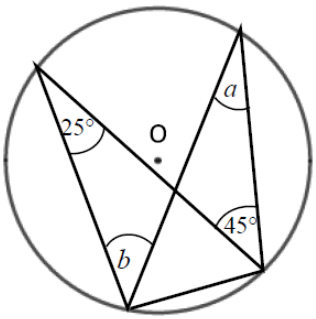
6) Estimate the shaded area using five strips of equal width.



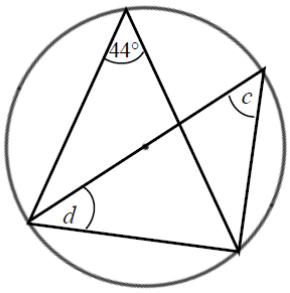
Section 10 Circle Theorems

Work out the size of each angle marked with a letter.
Give reasons for your answers.

a

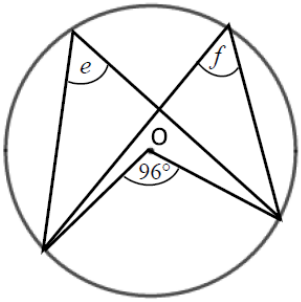


b



Hint
One of the angles is in a semicircle.

c



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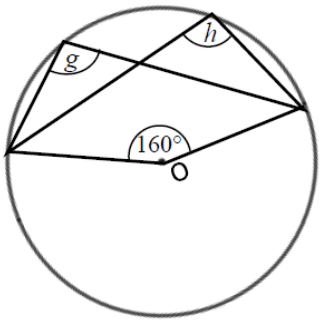
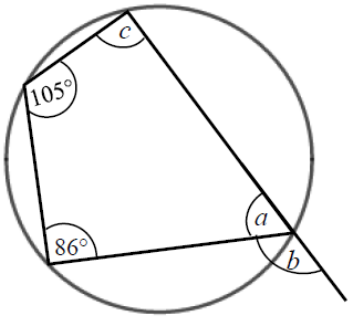
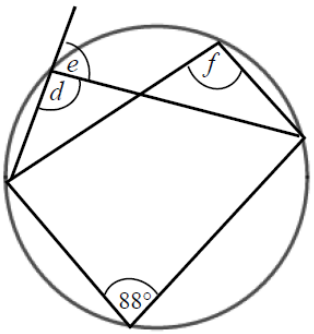


e



f)



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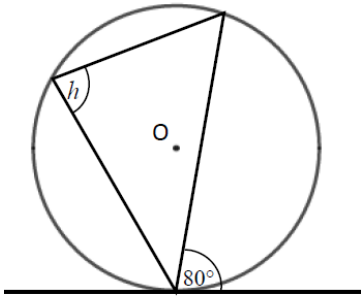
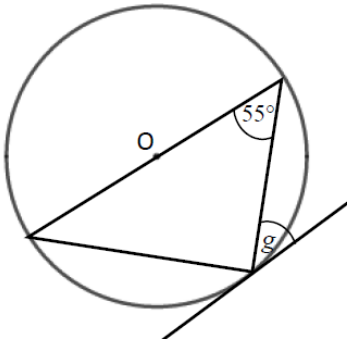


h



i) Prove the alternate segment theorem.

Answers

Section 1

1. Expand and Simplify where possible

a) $6x - 3$ b) $-3xy + 2y^2$ **c)** $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$

d) $8x - 6 - 3x - 5 = 5x - 11$ **e)** $2p - 7p^2$ **f)** $6b^2$ g)**5** $y - 4$

h) $-1 - 2m$ i) $5p^3 + 12p^2 + 27p$ j) $7x(3x - 5) = 21x^2 - 35x$

2.Factorise.

a) $2x^3y^3(3x - 5y)$ **b)** $7a^3b^2(3b^3 + 5a^2)$ **c)** $5x^2y^2(5 - 2x + 3y)$

3.Solve these Equations

a) 1 b) ½ c) 7 d) 15 e) 24/7 f) 3 g) 2

4.Solve these simultaneous equations using the **elimination method.**

a) $x = 3, y = -2$ b) $x = 6, y = -1$ c) $x = -2, y = 5$

5. Solve these simultaneous equations using the **substitution method.**

a) $x = -2, y = -7$ b) $x = \frac{1}{2}, y = 3$ c) $x = -2, y = -5$ d) $x = -2, y = 2\frac{1}{2}$

6. $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

7. Solve these pairs of simultaneous equations **graphically.**

a $x = 2, y = 5$ b) $x = -2, y = 2$ **c)** $x = 0.5, y = 0.5$ d) $x = -1, y = -2$

Section 2 Quadratic Expressions and Equations

1. Expand and simplify.

a) $x^2 + 5x - 14$ b) $10x^2 - 31x + 15$ c) $18x^2 + 39xy + 20y^2$ d) $16x^2 - 24xy + 9y^2$

e) $2x^2 - 2x + 25$ f) $x^2 - 1 - \frac{2}{x^2}$ g) $x^2 + 2 + \frac{1}{x^2}$

2. Factorise

a. $(x + 3)(x + 4)$ b. $(x - 5)(x - 6)$ c. $(x - 9)(x + 2)$ d. $(6x - 7y)(6x + 7y)$

e. $2(3a - 10bc)(3a + 10bc)$ f) $(x - 1)(2x + 3)$

3. Simplify the algebraic fractions.

a) $\frac{2(x+2)}{x-1}$ b) $\frac{x+2}{x}$ c) $\frac{x}{x+5}$ d) $\frac{x}{x-5}$ e) $\frac{3x+4}{x+7}$

f) $(x+5)$ g) $\frac{4(x+2)}{x-2}$ h) $\frac{2x+3}{3x-2}$

4. Solving quadratic equations

1. Solve these equations by **factorising**

a) $x = 0$ or $x = -\frac{2}{3}$ b) $x = -1$ or $x = 4$ c) $x = -\frac{1}{2}$ or $x = 4$ d) $x = -2$ or $x = 5$

e) $x = -5$ or $x = 5$ f) $x = -3$ or $x = 2\frac{1}{2}$

2. Solve these equations **by completing the square**

a) $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ b) $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$

c) $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$ d) $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$

e) $x = \frac{-3+\sqrt{23}}{2}$ or $x = \frac{-3-\sqrt{23}}{2}$

3. Solve these equations **by formula**

a) $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

b) $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

4. Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a) $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

b) $x = -1\frac{2}{3}$ or $x = 2$

5. Quadratic Simultaneous Equations

a) $x = 1, y = -2$
$x = 2, y = -1$

b) $x = 3, y = 4$
$x = 2, y = 1$

c) $x = -2, y = -4$
$x = 2, y = 4$

d) $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$
$x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

6. Solve these pairs of simultaneous equations **graphically**.

a) $x = 1, y = 0$ and $x = 4, y = 3$

b) $x = -2, y = 7$ and $x = 2, y = -5$

c) $x = -3, y = 4$ and $x = 4, y = -3$

Section 3 Inequalities

Solve these **linear inequalities**.

a) $x \leq -1$ b) $x > -\frac{7}{2}$ c) $x < -15$ d) $-1 \leq x < 5$ e) $x < -6$

f) $x < \frac{3}{2}$ g) $x > 5$ (which also satisfies $x > 3$)

Quadratic Inequalities

a) $-7 \leq x \leq 4$ b) $x \leq -2$ or $x \geq 6$ c) $-3 \leq x \leq 4$ d) $2 < x < 2\frac{1}{2}$

e) $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$

Section 4 Equation of Straight Lines

1.Find the gradient and the *y*-intercept of the following equations.

a) $m = -\frac{1}{2}$, $c = -7$ b) $m = -1$, $c = 5$ c) $m = \frac{2}{3}$, $c = -\frac{7}{3}$ or $-2\frac{1}{3}$

2.

a) $x + 2y + 14 = 0$ b) $2x - y = 0$ c) $2x - 3y + 12 = 0$

3) $y = 4x - 3$ 4) $y = -\frac{2}{3}x + 7$ 5) $y = 2x - 3$

6) $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the *y*-intercept is 3.

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

Parallel and Perpendicular Lines

1a) $y = 3x - 7$ b) $y = -\frac{1}{2}x$

2a) $y = -\frac{1}{2}x + 2$ b) $y = -4x + 35$

3a) $y = -\frac{1}{2}x$ b) $y = 2x$

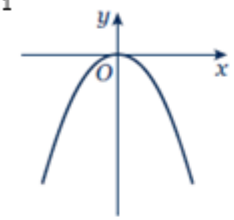
4a) Perpendicular b) Neither **c)** Parallel

5a) $x + 2y - 4 = 0$ **b)** $x + 2y + 2 = 0$ **c)** $y = 2x$

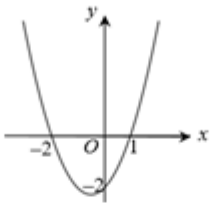
Section 5 Sketching Graphs

Quadratic Graphs

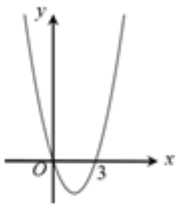
1`



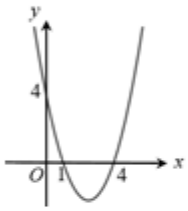
2a)



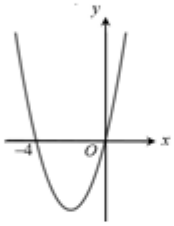
2b)



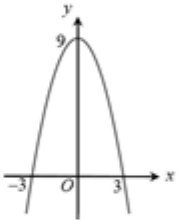
2c)



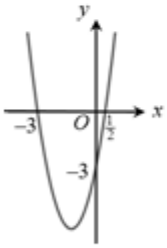
2d)



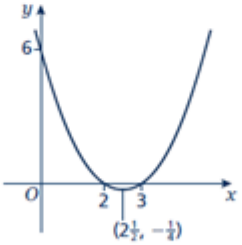
2e)



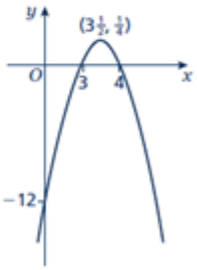
2f)



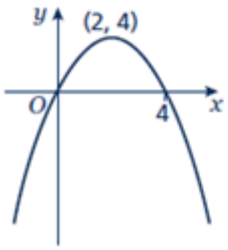
3a)



3b)



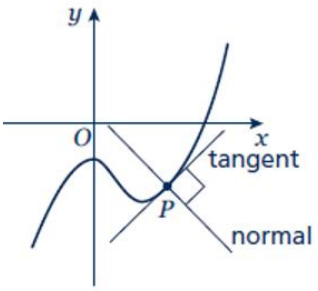
3c)



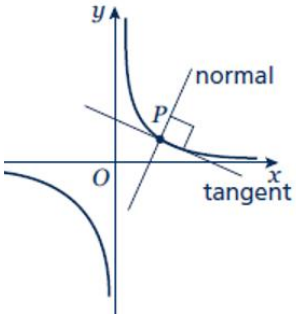
Cubic and Reciprocal Graphs

i – C ii – E iii – B iv – A v – F vi – D

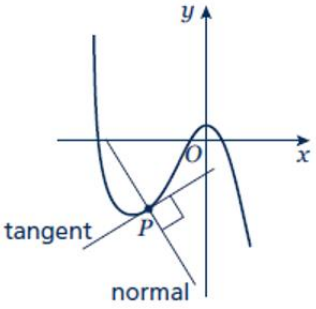
b ii



iv

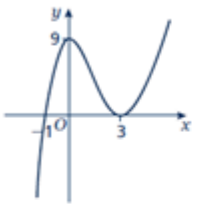


vi

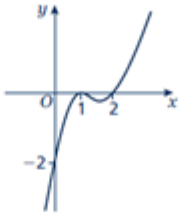


5. Sketch the following graphs

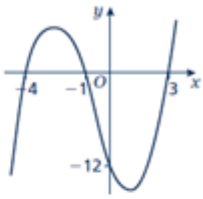
a)



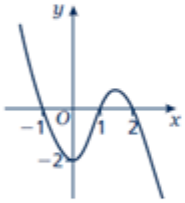
b)



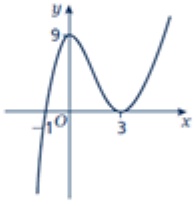
c)



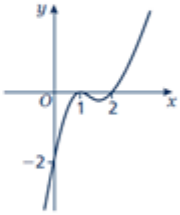
d)



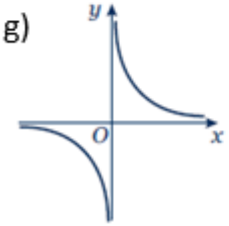
e)



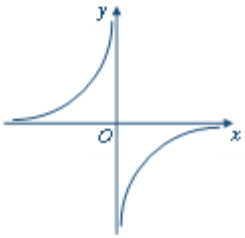
f)



g)

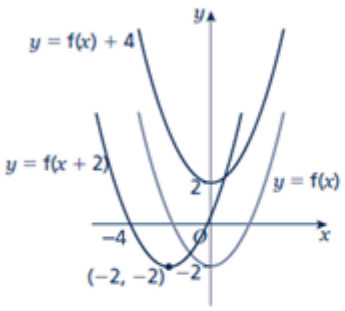


h)

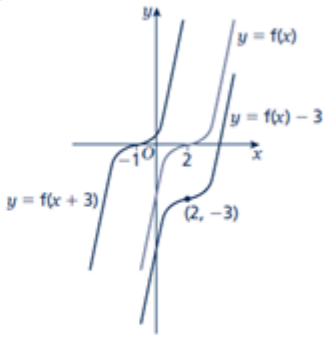


Transformations of Graphs

6)



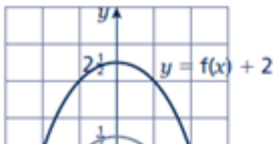
7`



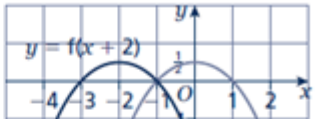
8) C_1: $y = f(x - 90°)$
C_2: $y = f(x) - 2$

9) C_1: $y = f(x - 5)$
C_2: $y = f(x) - 3$

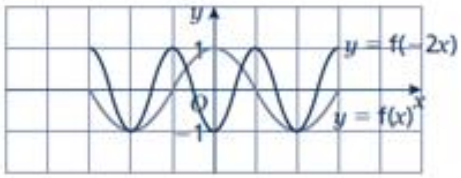
10a)



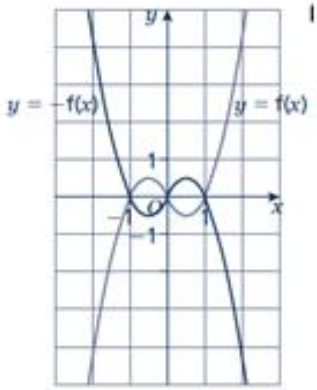
10b)



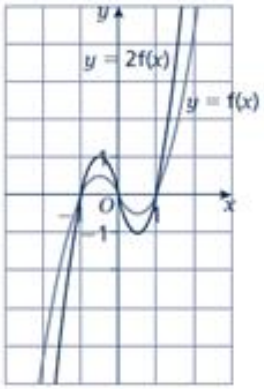
13)



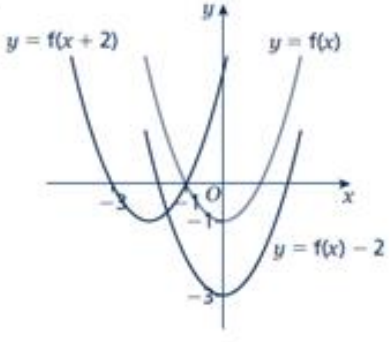
13a)



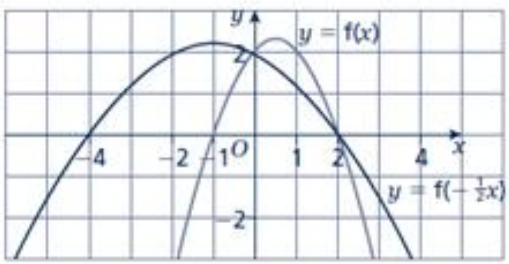
13b)



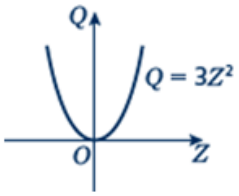
14)



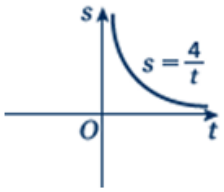
15)



Section 6 Proportion

1) £77 **2a)** $Q = 3Z^2$ 2b) 

2c) 75 **2d)** ±10

3) 5 4a) $s = \frac{4}{t}$ 4b) 

4c) 4 5) 1

6a) 0.1 **5b)** 0.1

Section 7 Indices and Surds

1.Evaluate.

a) 1 b) 4 c) 343 d) $\frac{1}{36}$

2. Simplify.

a) $\frac{3x^3}{2}$ b) $2x^6$ c) $y^{\frac{1}{2}}$ d) x

3. Evaluate.

a) $\frac{1}{9}$ b) $\frac{1}{4}$ c) $\frac{16}{9}$

4.Write the following as a single power of x.

a) $x^{\frac{2}{5}}$ b) $x^{-\frac{1}{3}}$ c) $x^{-\frac{2}{3}}$

5. Write the following without negative or fractional powers.

a) $\frac{1}{x^3}$ b) $\sqrt[5]{x^2}$ c) $\frac{1}{\sqrt[4]{x^3}}$

6. Write the following in the form ax^n.

a $5x^{\frac{1}{2}}$ **b** $2x^{-3}$ **c** $\frac{1}{3}x^{-4}$

d $2x^{-\frac{1}{2}}$ **e** $4x^{\frac{1}{3}}$ **f** $3x^0$

7. Write as sums of powers of x.

a $x^3 + x^{-2}$ **b** $x^3 + x$ **c** $x^{-2} + x^{-7}$

8. Simplify.

8a) $15\sqrt{2}$ b) $\sqrt{3}$ c) $5\sqrt{3}$

9a) $10\sqrt{5}-7$ b) $26-4\sqrt{2}$ c) $x-y$

10 a) $\frac{2\sqrt{7}}{7}$ b) $\frac{\sqrt{3}}{3}$ c) $\frac{1}{3}$ d) $\frac{2(4-\sqrt{3})}{13}$ e) $\frac{6(5+\sqrt{2})}{23}$

f) $3+2\sqrt{2}$ g) $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Section 8 Pythagoras and Trigonometry

1.

a $18\sqrt{13}$ mm **b** $2\sqrt{145}$ mm

c $42\sqrt{2}$ mm **d** $6\sqrt{89}$ mm

2) 95.3 mm 3) 64.0 km 4) $3\sqrt{5}$ units 5) $4\sqrt{3}$ cm

6) 2.80 cm 7) 47.0° 8) 20.4°

Cosine Rule

9a) 70.8 mm 9b) 122.9° 10**a)** 13.7 cm 10**b)** 76.0°

Sine Rule

11) 45.2 mm 12) 53.6° 13) 693 mm^2 14) 15.3 cm

Section 9 Area and Volume

1) $V = x^3 + \frac{17}{2}x^2 + 4x$ 2) 60 cm^3 3) 21.4 cm 4) 32 : 9 5) $r = \sqrt[3]{36x}$

Area under graph

1) 34 $units^2$ 2) 149 $units^2$ 3) $26\frac{7}{8}$ $units^2$ 4) 56 $units^2$ 5) 35 $units^2$

6) $6\frac{1}{4}$ $units^2$

Section 10 Circle Theorems

a) a = 25°, angles in the same segment are equal.
b = 45°, angles in the same segment are equal.

b) c = 44°, angles in the same segment are equal.
d = 46°, the angle in a semicircle is 90° and the angles in a triangle total 180°.

c) e = 48°, the angle at the centre of a circle is twice the angle at the circumference.
f = 48°, angles in the same segment are equal.

d) g = 100°, angles at a full turn total 360°, the angle at the centre of a circle is twice the angle at the circumference.
h = 100°, angles in the same segment are equal.

e) a = 75°, opposite angles in a cyclic quadrilateral total 180°.
b = 105°, angles on a straight line total 180°.
c = 94°, opposite angles in a cyclic quadrilateral total 180°.

f) d = 92°, opposite angles in a cyclic quadrilateral total 180°.
e = 88°, angles on a straight line total 180°.
f = 92°, angles in the same segment are equal.

g) h = 80°, alternate segment theorem.

h) g = 35°, alternate segment theorem and the angle in a semicircle is 90°.

i)

Angle BAT = x.

Angle OAB = 90° − x because the angle between the tangent and the radius is 90°.

OA = OB because radii are equal.

Angle OAB = angle OBA because the base of isosceles triangles are equal.

Angle AOB = 180° − (90° − x) − (90° − x) = $2x$ because angles in a triangle total 180°.

Angle ACB = $2x \div 2 = x$ because the angle at the centre is twice the angle at the circumference.